Thermal Storage Life of Solid-Propellant Motors

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The storage life of a solid-propellant rocket motor subjected to environmental temperature variations has been calculated with the aid of probabilistic mechanics. Viscoelastic effects, chemical aging, cumulative damage, and statistical variation in mechanical properties were examined.

Nomenclature

a	$=\cos 2\pi t/360\times 24$
a_T	= viscoelastic shift function
A_1, A_2	= constants of viscoelastic strength
B	= parameter of aging factor
B_d	= parameter of cumulative damage
$C, \bar{C}$	= parameters of cumulative damage, mean value
C_1, C_2	= parameters of shift function (C_1, dimensionless; C_2, °C)
d_i	= damage at *i*th stress level
D	= daily damage
$E(\cdot)$	= expected value
$E_r(t), E'(t), E_i$	= relaxation moduli, N/mm²
E_∞	= rest modulus, N/mm²
$E'(\omega), E''(\omega)$	= complex moduli, N/mm²
f_i	= frequency of occurrence of *i*th stress level
$f_S(s), f_{S_y}(s), f_{S_d}(s)$	= probability density functions, for annual, for daily peak stresses, respectively
$F_R(r)$	= probability function of strength
$g(s)$	= function of stress
L_i, L_n	= reliability on *i*th day, on *n*th day, respectively
P, P_f, P_{fi}	= probability, probability of failure, on *i*th day, respectively
r, r_c	= specific value of strength, characteristic value of Weibull distribution, respectively, N/mm²
$R, \bar{R}, R_0$	= strength variable, mean, and initial, respectively, N/mm²
$S, S(t), S_m', S_d$	= stress variable, time dependent, daily mean, and daily peak, respectively, N/mm²
s	= specific value of stress, N/mm²
t, t_1, t_2	= time
t_i, t_{fi}, t_f, t_d	= time spent at *i*th stress level, time to failure at *i*th stress level, time to failure under variable stress, and time to damage, respectively, h
t_p	= period, h
T, T_0, T_y	= temperature, reference, and annual peak, respectively, °C
β	= Weibull shape parameter
$\beta_1, \beta_2, \beta_R$	= aging parameters
$\Gamma(\cdot)$	= gamma function
δ_R, δ_c	= coefficient of variation for strength and for damage, respectively
$\Delta, \Delta_d, \bar{\Delta}, \Sigma\Delta$	= damage, daily, average, and cumulative, respectively
$\epsilon(t), \epsilon_m, \epsilon_y, \epsilon_d$	= strain, mean, annual peak, and daily peak, respectively
$\eta_1, \eta_2, \eta_R, \eta_E$	= aging parameters, for strength and for modulus, respectively
μ_y, μ_s	= mean temperature and mean stress, respectively
$\sigma_r, \sigma_{\eta_r}, \sigma_c, \sigma_\Delta, \sigma_R$	= standard deviation of strength, aging, damage parameter, damage, and aged strength, respectively
τ_i	= relaxation time, h
$\omega, \omega_d, \omega_y$	= circular frequency, of a day, and of a year, respectively

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Introduction

IN a 1979 paper[1] the authors presented a basic methodology for the calculation of the storage life of a solid-propellant motor in a random thermal environment. Heat-transfer and thermal stress analysis of a long, circular bore, case-bonded cylinder consisting of an elastic material was developed in detail. Probabilistic failure criteria were used. The present paper is a continuation of that work. Here the motor is considered to be viscoelastic and subject to chemical aging and stress-dependent cumulative damage, i.e., the statistically variable initial strength of the material is degraded by variable amplitude thermal stresses. The motor is considered to have reached its design service life when the probability that the environmental stresses exceed the degraded strength becomes excessively high.

Viscoelastic Analysis

For the sake of brevity the transient heat-transfer relations and the frequency response functions for elastic thermal stresses will not be developed here.[1] The relations will be modified, however, for viscoelastic behavior.

The time-temperature sensitivity of solid-propellant materials is usually characterized by a reduced-time-dependent relaxation modulus and reduced-time-dependent strength and strain capacity. The master curve of the relaxation modulus for a propellant is usually expanded into a Prony series and may be written as

$$E_r(t) = E_\infty + E'(t) \tag{1}$$

where

$$E'(t) = \sum_{i=1}^{15} E_i e^{-t/\tau_i a_T} \tag{2}$$

and $E'(t) \to 0$ as $t \to \infty$, where E_∞ is the equilibrium modulus, E_i and τ_i the moduli and relaxation times of parallel Maxwell elements, and a_T the viscoelastic shift function,

$$\log a_T = \frac{-C_1(T-T_0)}{C_2+(T-T_0)} \tag{3}$$

Because in storage thermal loads are essentially cyclic in nature, it is expedient to convert the time-dependent relaxation modulus into the frequency-dependent complex modulus representation. This is accomplished by performing sine and cosine Fourier transformations, as indicated by Christensen,[2] which yield a real and an imaginary term

$$E'(\omega) = E_\infty + \sum_{i=1}^{15} \frac{E_i \omega^2 a_T^2 \tau_i^2}{1+\omega^2 \tau_i^2 a_T^2} \tag{4}$$

and

$$E''(\omega) = \sum_{i=1}^{15} \frac{E_i \omega a_T \tau_i}{1+\omega^2 \tau_i^2 a_T^2} \tag{5}$$

In terms of the period of cyclic loading, $\omega = 2\pi/t_p$, Eqs. (4) and (5) may be rewritten as

$$E'(\omega) = E_\infty + (2\pi)^2 \sum_{i=1}^{15} \frac{E_i a_T^2 \tau_i^2}{t_p^2 + 4\pi^2 \tau_i^2 a_T} \tag{6}$$

$$E''(\omega) = 2\pi \sum_{i=1}^{15} \frac{t_p E_i \tau_i a_T}{t_p^2 + 4\pi^2 \tau_i^2 a_T^2} \tag{7}$$

The relaxation modulus and the two parts of the complex modulus are presented in Fig. 1 for a typical propellant. It is seen that for values of $\log t/a_T$ greater than -8, the loss modulus becomes negligibly small. Because the range of temperatures encountered under storage conditions is $-40 < T < 54$ °C, and the periods of the cyclic components are $1 < t_p < 8760$ h, the values of $\log t_p/a_T$ will always be greater than -8. Hence neglecting $E''(\omega)$ and using the storage modulus alone will result in little error.

The major contributions to storage strains are produced by the annual mean temperature and the two cyclic components (seasonal and diurnal); consequently, strain can be expressed as

$$\epsilon(t) = \epsilon_m + \epsilon_y \sin\omega_y t + \epsilon_d \sin\omega_d t \tag{8}$$

and the corresponding stress as

$$S(t) = \epsilon_m E(t) + \epsilon_y [E'(\omega_y) + iE''(\omega_y)] e^{i\omega_y t} + \epsilon_d [E'(\omega_d) + iE''(\omega_d)] e^{i\omega_d t} \tag{9}$$

where the first term represents the stress relaxation under constant mean strain and the two cyclic components will result in cyclic stresses of constant amplitudes and phase angles.

The frequency response functions developed in Ref. 1 for elastic stresses may now be reused by replacing the elastic modulus with the complex modulus for cyclic components and with the relaxation modulus for constant stresses.

Tests conducted at various temperatures and several constant strain rates have been reported in the literature in the form of failure strength and strain at failure vs reduced-time curves. The master curve for the mean viscoelastic strength $\bar{R}$ may be represented by an equation of the type

$$\log\left[\bar{R}\frac{T_0}{T}\right] = A_1 + A_2 \log(t_p/4a_T) \tag{10}$$

where a_T is the shift function of Eq. (3).[4]

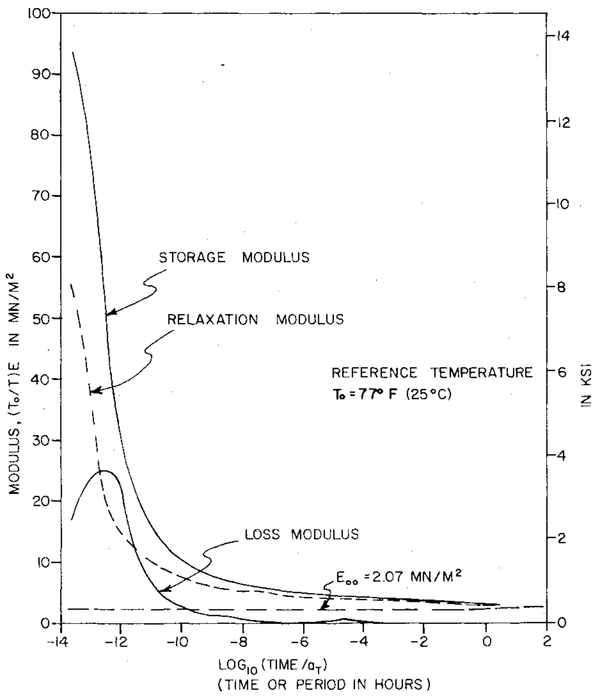

Fig. 1 Relaxation and complex moduli.

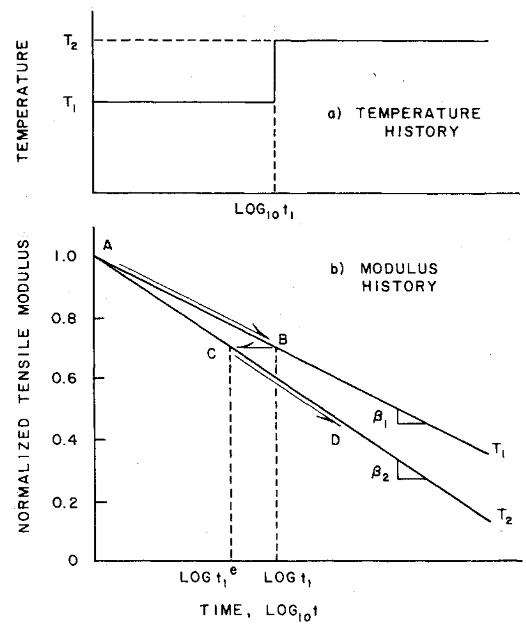

Fig. 2 Variable temperature aging model.

The scatter of the data is characterized by a coefficient of variation δ_R.

Based on the concepts of "increasing failure rates" for a "wear-out" type of process such as ultimate strength, the Weibull distribution has been proposed as most suitable for the description of the statistical variations of strength.[3]

The probability that the strength is less than a given value is consequently written as

$$P[R \le r] = F_R(r) = 1 - \exp[-(r/r_c)^\beta] \tag{11}$$

where r_c is a "characteristic value" or location parameter of the Weibull distribution and β is the shape parameter defined in terms of the mean $\bar{R}$ and standard deviation σ_r as

$$r_c = \bar{R}/\Gamma(1+1/\beta) \tag{12}$$

and

$$\sigma_r^2 = r_c^2 [\Gamma(1+2/\beta) - \Gamma^2(1+1/\beta)] \qquad (13)$$

The value of β may be estimated from the coefficient of variation as

$$\beta \sim 1.2/\delta_R \qquad (14)$$

Aging

Solid-propellant materials in addition to being viscoelastic are also subject to 1) time- and temperature-dependent chemical aging, i.e., changes in their physical and thermal parameters in an unloaded condition; and 2) a separate load-, time-, and temperature-dependent deterioration of their strength and strain capacities commonly referred to as cumulative damage.

Aging effects at constant temperatures are responsible for the hardening or softening of rubbery materials, that is, changes in their viscoelastic moduli or the reduction of their strength and strain capacities even in the absence of applied loads. Layton[4] and Cost[5] have found that the ratio $\eta(T,t)$ of the current value of a property to its initial value is proportional to the logarithm of time. In the case of the strength

$$\eta_R(T,t) = \frac{R(T,t)}{R(T,0)} = 1 - \beta_R(T)\log t \qquad (15)$$

where the coefficient $\beta_R(T)$ is an exponentially decreasing function of the absolute temperature T. Hence

$$\beta_R(T) = A_R e^{-B/T} \qquad (16)$$

Of course, a similar expression may be written for the deterioration factor of initial modulus η_E in terms of different constants A and B.

To calculate the mean and standard deviation of an aged physical property the mathematics of statistical expectations are used

$$R = \eta_R R_0 \qquad (17)$$

The expected or average value of R, $E(R) = \bar{R}$ can be expressed in terms of the expected values $E(\eta_R) = \bar{\eta}_R$ and $E(R_0) = \bar{R}_0$ as

$$E(R) = E(\eta_R) \times E(R_0) = \bar{\eta}_R \bar{R}_0 \qquad (18)$$

Similarly, the expected value of R^2 is

$$E(R^2) = E(\eta_R^2) \times E(R_0^2) \qquad (19)$$

in terms of the standard deviations

$$E(R^2) = (\sigma_R^2 + \bar{R}^2) = (\sigma_{\eta_R}^2 + \bar{\eta}_R^2)(\sigma_{R_0}^2 + \bar{R}_0^2) \qquad (20)$$

Hence

$$\sigma_R^2 = \sigma_{R_0}^2 (\bar{\eta}_R^2 + \sigma_{\eta_R}^2) + \bar{R}_0^2 \sigma_{\eta_R}^2 \qquad (21)$$

Consequently the standard deviation of the aged strength (modulus) can be calculated from the means and standard deviations of the initial strength (modulus) and the aging factor.

It is evident that aging is accelerated at higher temperatures and consequently more aging will occur during the summer months and in hot climate storage areas. Variable temperature aging will be calculated based on the concept of reduced time as suggested by Cost.[5] Figure 2 shows the basic steps of the method. If the propellant is aged at temperature T_1 for a period of t_1 days the aging factor becomes equal to

$$\eta_1 = 1 - \beta_1 \log t_1 \qquad (22)$$

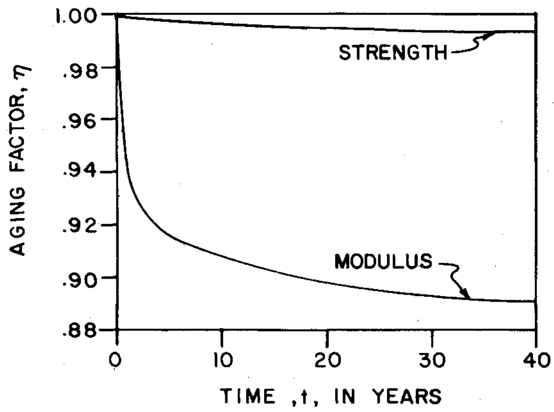

Fig. 3 Aging effects on modulus and strength.

Table 1 Physical parameters

Stress-free temperature, $U_f = 71\,°C$
Mean temperature, $\mu_y = 23\,°C$
Average seasonal temperature amplitude, $T_y = 12.2\,°C$
Diffusivity of air, $\alpha_1 = 1.915 \times 10^{-5}$ m²/s
Diffusivity of casing, $\alpha_3 = 8.770 \times 10^{-6}$ m²/s
Thermal conductivity of air, $k_1 = 2.45 \times 10^{-2}$ W/m·°C
Thermal conductivity of propellant, $k_2 = 4.09 \times 10^{-1}$ W/m·°C
Thermal conductivity of casing, $k_3 = 2.53 \times 10^{1}$ W/m·°C
Radius of cavity, $a = 0.064$ m
Outer radius of propellant, $b = 0.201$ m
Outer radius of casing, $c = 0.203$ m
Poisson's ratio for propellant, $\nu_2 = 0.49$
Poisson's ratio for casing, $\nu_3 = 0.25$
Coefficient of thermal expansion for propellant, $\bar{\alpha}_2 = 3.3 \times 10^{-5}$ m/m/°C
Coefficient of thermal expansion for casing, $\bar{\alpha}_3 = 3.6 \times 10^{-6}$ m/m/°C
Modulus of elasticity for propellant, $E_\infty = 2 \times 10^6$ N/m²
Modulus of elasticity for casing, $E_3 = 2.07 \times 10^{11}$ N/m²
Shift factor parameters, $C_1 = -7$, $C_2 = 150$ K, $T_0 = 300$ K
Strength parameters, $A_1 = 2$, $A_2 = -0.1$, $\delta_R = 0.10$
Aging parameters for modulus, $A_E = 4 \times 10^5$, $B_E = 5 \times 10^3$, $\sigma_E = 0.07$
Aging parameters for strength, $A_R = 1 \times 10^{10}$, $B_R = 9 \times 10^3$, $\sigma_R = 0.05$
Damage parameters, $C = 2 \times 10^{51}$, $B_d = 9$, $\delta_c = 0.90$

where $\beta_1 = Ae^{-B/T_1}$. The same aging factor may be obtained at a different temperature T_2 in time t_1', called equivalent time

$$\eta_1 = 1 - \beta_2 \log t_1' = 1 - \beta_1 \log t_1 \qquad (23)$$

Hence the equivalent time during which the same aging parameter is reached at T_2 becomes equal to

$$t_1' = t_1^{\beta_1/\beta_2} \qquad (24)$$

If aging is now continued at T_2 for an additional time Δt the total aging time t_2 will be the sum

$$t_2 = t_1' + \Delta t \qquad (25)$$

The process is then repeated for other temperatures.

Because the aging process at practical service temperatures is relatively slow, it has been found that diurnal temperature variations have an insignificant effect on aging factors. As a consequence only seasonal thermal changes have been included in aging calculations. Aging factors are evaluated for the average daily temperature

$$T = \mu_y - T_y \cos(t) \qquad (26)$$

where the temperatures are measured in degrees Kelvin, T_y is the mean amplitude of the seasonal cycle, μ_y the annual average temperature, and t the day of the year. When t is measured in days and a year is assumed to consist of 360 days, the argument of the cosine term in Eq. (26) is essentially in degrees of angle.

Aging calculations are carried out on strength and modulus values previously adjusted for viscoelastic variations. Figure 3 shows aging effects at a typical southwest United States storage location described in detail in Ref. 1. The parameters of Eq. (16) are listed in Table 1.

Cumulative Damage

The linear cumulative damage rule proposed by Palmgren[6] and Miner[7] has been used extensively in the aerospace industry. According to this rule the damage produced in a unit of time spent at a particular stress level S_i is inversely proportional to the time t_{fi} required to produce failure in the material at that stress level

$$d_i = 1/t_{fi} \tag{27}$$

and when a mixture of n stress levels is present each for a time t_i, the total damage D becomes equal to

$$D = \sum_{i=1}^{n} \frac{t_i}{t_{fi}} \tag{28}$$

Experiments conducted by Bills[8] indicate that the linear cumulative damage rule may be useful in the prediction of service life of solid propellants. It has also been shown that a power function will describe the relationship between applied constant stress and reduced time to failure,

$$t_f/a_T = CS - B_d \tag{29}$$

where C and B_d are material parameters listed in Table 1, a_T the viscoelastic shift factor, and S the stress, adjusted for aging and viscoelastic effects. When plotted on double logarithmic scales such a relation forms a straight line as shown in Fig. 4.

Failure time t_f is a statistical variable and has been observed to be afflicted by a significant amount of dispersion. The statistical character of the $(t_f - S)$ relations has been assumed to result in a series of parallel lines on double logarithmic scales. Hence damage lines at various probability levels differ from each other only in the value of the parameter C. The coefficient of variation of t_f is equivalent to the coefficient of variation δ_c of the parameter C.

If the total time, under a random set of stresses is denoted by t_d, of which a fraction f_i is spent at stress level S_i, Eq. (28) may be rewritten as

$$D = \sum_{i=1}^{n} \frac{f_i t_d}{t_{fi}} \tag{30}$$

For continuously varying stress levels the summation is replaced by integration and the fraction f_i becomes the probability density function $f_S(s)$. Substituting Eq. (29) into Eq. (30)

$$D = t_d \int \frac{f_S(s)\,ds}{a_T CS - B_d} = t_d \times \Delta \tag{31}$$

results with the integral denoted by Δ. To perform the integration indicated in Eq. (31) the probability density function of stresses has to be developed.[9]

During a diurnal cycle of amplitude S_d and daily mean S'_m as shown in Fig. 5, stress can be described by a sinusoidal relation

$$S = S'_m + S_d \sin \frac{\pi(t-6)}{12} \tag{32}$$

The proportion of time spent between s and $s+ds$ during one complete cycle which is equivalent to the probability that stress is between those limits is calculated next.

Differentiating Eq. (32)

$$ds = S_d \frac{\pi}{12} \cos \frac{\pi(t-6)}{12}\, dt \tag{33}$$

Solving Eq. (33) for dt, using Eq. (32) and the trigonometric identity $\sin^2\alpha + \cos^2\alpha = 1$,

$$dt = \frac{ds}{S_d \frac{\pi}{12}\sqrt{1-\left(\frac{S-S'_m}{S_d}\right)^2}} = \frac{ds}{\frac{\pi}{12}\sqrt{S_d^2 - (S-S'_m)^2}} \tag{34}$$

The proportion of time per 24 h cycle spent between s and $s+ds$ is

$$\frac{2dt}{24} = P[s<S<s+ds] = f_s(s)\,ds = \frac{ds}{\pi\sqrt{S_d^2-(S-S'_m)^2}} \tag{35}$$

Therefore, the density function of stresses during a $t_d = 24$ h cycle becomes

$$f_S(s) = \frac{1}{\pi\sqrt{S_d^2-(S-S'_m)^2}} \tag{36}$$

When this density function is substituted into Eq. (31), the mean damage per daily cycle can be calculated as

$$\Delta_d = \frac{t_d}{C\pi a_T} \int_{S'_m - S_d}^{S'_m + S_d} \frac{ds}{S^{-B_d}\sqrt{S_d^2-(S-S'_m)^2}} \tag{37}$$

Equation (37) can be integrated numerically. The daily variation of average damage is shown in Fig. 6, and the average accumulated damage over a 10 yr period is presented in Fig. 7.

It should be noted that the parameter C in the $S-t_f$ relationship of Eq. (29) is a statistically variable quantity

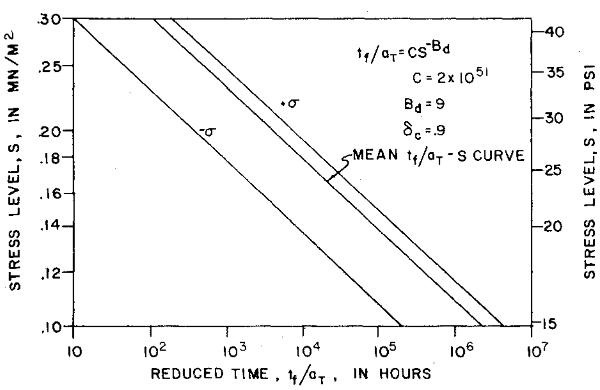

Fig. 4 Stress failure-time relation.

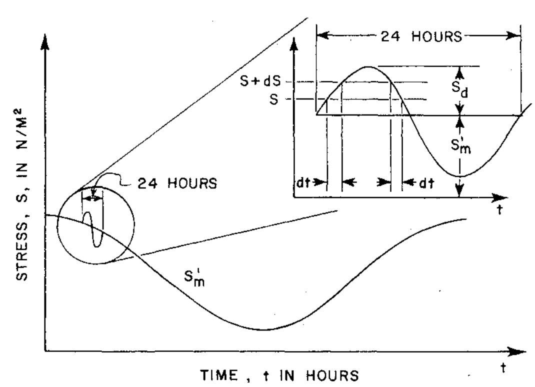

Fig. 5 Simplified temperature variation.

which will result in similar statistical variations in the daily damage. Writing Eq. (37) in a more compact form

$$\Delta_d = g(s)/C \tag{38}$$

where $g(s)$ is the remainder of Eq. (37) after extracting C, and expanding the damage into a Taylor series around the mean value

$$\frac{g(s)}{C} = g(s)\left[\frac{1}{\bar{C}} - \frac{1}{\bar{C}^2}(C-\bar{C})\right] + \ldots \tag{39}$$

with second- and higher-order terms neglected, the difference

$$g(s)\left[\frac{1}{C} - \frac{1}{\bar{C}}\right] = \frac{g(s)}{\bar{C}^2}(C-\bar{C}) \tag{40}$$

is obtained. The expected value of the square of the left side is the variance of the damage

$$E\left\{g^2(s)\left[\frac{1}{C} - \frac{1}{\bar{C}}\right]^2\right\} = E\left[\frac{g(s)^2}{\bar{C}^4}(\delta_c\bar{C})^2\right] \tag{41}$$

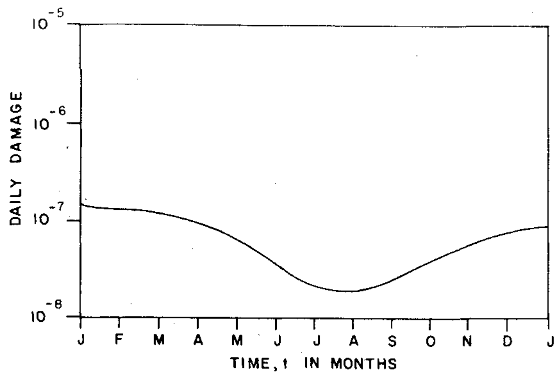

Fig. 6 Daily variation of average damage.

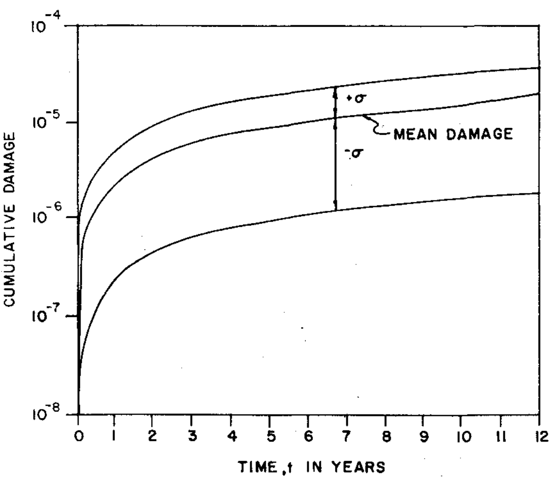

Fig. 7 Accumulated damage.

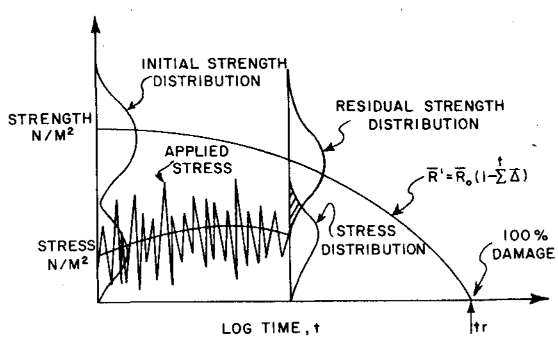

Fig. 8 Stress and residual strength variations with time.

while the right side is related to the variance of the parameter C. Equation (41) can be written as

$$\sigma_\Delta^2 = \frac{g^2(s)}{\bar{C}^4}\sigma_c^2 = \frac{g^2(s)}{\bar{C}^4}(\delta_c\bar{C})^2 \tag{42}$$

Because the mean damage $\bar{\Delta} = gs)\bar{C}$ the standard deviation of damage becomes, in first approximation, equal to

$$\sigma_\Delta = \delta_c\bar{\Delta} \tag{43}$$

Combined Failure Due to Cumulative Damage and Load Excursion

In previous papers[1,10] failure produced by excessive thermal loads has been discussed separately from cumulative damage. On the one hand it has been assumed that the motor fails if thermal stresses in excess of the initial strength are applied, while on the other hand cumulative damage failure was postulated to take place when the damage ratio increased to unity.

It is, however, evident that the two concepts must be combined. Cumulative damage results in a reduction of material strength and an overload condition may consequently occur sooner than under the no-damage assumption. The process is illustrated in Fig. 8. The initial distribution of strength changes due to viscoelastic effects, aging, and cumulative damage and hence the probability of an overload condition increases gradually.

Assuming that damage accumulation is equivalent to a linear strength reduction,[11] the mean residual strength $\bar{R}'$ of the material at any time can be characterized as

$$\bar{R}' = \bar{\eta}_R\bar{R}_0(1-\Sigma\bar{\Delta}) = \bar{R}(1-\Sigma\bar{\Delta}) \tag{44}$$

where $\bar{R}_0$ is the mean virgin strength adjusted for viscoelastic effects, $\bar{\eta}_R$ the average strength aging factor, and $\Sigma\bar{\Delta}$ the mean cumulative damage ratio, i.e., the sum of daily damages obtained from Eq. (37).

The standard deviation of the residual strength is obtained by calculating the expected value of $(R')^2$,

$$\begin{aligned} E[(R')^2] &= E[(\bar{R}'^2 + \sigma_{R'}^2) = E[R^2]\times E[(1-\Sigma\Delta)^2] \\ &= E[R^2]\times E[1-2\Sigma\Delta + (\Sigma\Delta)^2] \\ &= (\bar{R}^2+\sigma_R^2)\,[1-2\Sigma\bar{\Delta} + (\Sigma\bar{\Delta}^2 + \sigma_\Delta^2)] \end{aligned} \tag{45}$$

and since $\bar{R}' = \bar{R}(1-\Sigma\bar{\Delta})$,

$$\sigma_{R'}^2 = \bar{R}^2\,[\delta_c^2(1+\delta_R^2)(\Sigma\bar{\Delta})^2 + \delta_R^2(1-\Sigma\bar{\Delta})^2] \tag{46}$$

The standard deviation σ_R is obtained in terms of the standard deviation of theoretical strength and that of the aging factor.

Once the mean and standard deviation of the residual strength has been calculated on a daily basis, the stress-strength interference principle[1,10] will be employed. Probability of failure P_f due to the combined effects of damaged strength and overstress is defined as

$$P_f = P[R' \leq S] = P[(R'-S) \leq 0] \tag{47}$$

where R' is the residual strength adjusted for aging, viscoelastic effects, and cumulative damage and S the applied thermal stress also modified to account for aging and viscoelastic variations in the complex or relaxation modulus.

The probability density function of the difference of two random variables is a convolution integral, and the probability that this difference is less than zero may be written as[3]

$$P_f = \int_0^\infty F_R(s) f_S(s)\,\mathrm{d}s \tag{48}$$

where $F_R(s)$ is the probability that the strength R is less than the stress s and is given by Eq. (11) with parameter modified according to Eqs. (44) and (46). $f_S(s)$ is the density function of stress peaks detailed in Ref. 1 and modified for viscoelasticity and aging with the introduction of Eqs. (9), (17), and (21)

$$f_S(s) = \frac{1}{|a|}\int_{-\infty}^{\infty} f_{S_y}\left(\frac{s-\mu_s-s_d}{a}\right) f_{S_d}(s_d)\,\mathrm{d}s_d \qquad (49)$$

Here $a = \cos 2\pi t/360 \times 24$, μ_s is the mean stress, and s_d and s_y are the Raleigh distributed diurnal and seasonal peak stresses, respectively.

The daily reliability

$$L_i = 1 - P_{f_i} \qquad (50)$$

is progressively multiplied to yield the reliability of the motor after n days

$$L_n = \prod_{i=1}^{n} (1 - P_f) \qquad (51)$$

The process of the multiplication of L_i is carried out until a predetermined limiting, reliability value at the end of the service life is reached.

The progressive probability of failure for a typical propellant, whose properties are given in Table 1, has been calculated for environmental temperature conditions in the southwest United States and is shown in Fig. 9. It is seen that within seven years the probability of failure increases to 10^{-3}, i.e., one per thousand failures in seven years.

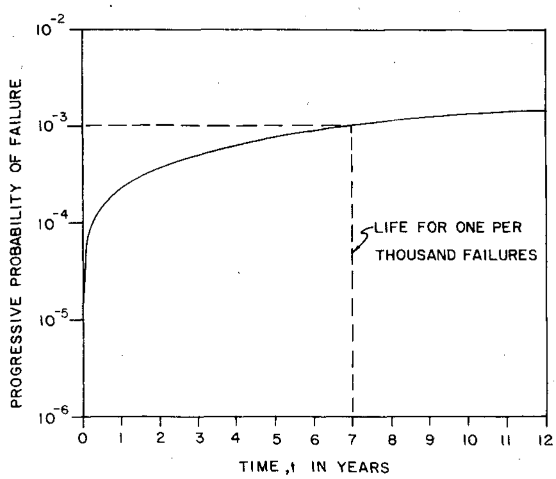

Fig. 9 Progressive probability of failure.

Discussion

In order to obtain a relatively simple solution to an otherwise complex problem, the thermal stress analysis of the motor was based on the assumption of plane strain conditions. Hence the length of the structure is not specified though lateral geometry is considered. As a consequence tangential bore stresses are the same along the full length of the cylinder and would, theoretically, result in a long crack when the strength of the material is exceeded. This assumption leads to conservative estimates for the probability of failure because, in actual motors, cracks are localized.

In a subsequent, finite-element analysis the length as well as complex bore geometries will be introduced. Should the strength of the material prove to be volume sensitive, a different analysis based on the weakest link hypothesis and the Weibull distribution would also have to be introduced.

The statistical distribution of propellant strength is usually based on relatively few experimental results. In order to obtain definite information about the lower tail of the distribution, that is, for failure probabilities of the order of 10^{-6}, 1×10^6 specimens would have to be tested. Because such a test program would be impractical, various distribution functions of strength were assumed and of these the most conservative, the Weibull distribution, was used. Should further statistical information become available, other distributions could be introduced.

Several effects, namely aging, viscoelasticity, and cumulative damage, were combined in the above life prediction methodology. For the particular propellant and storage location chosen, aging and cumulative damage are insignificant as can be seen from Figs. 3 and 7. The viscoelastic modulus approaches the rest modulus E_∞ (Fig. 1) and is nearly constant. Hence an elastic analysis with E_∞ would produce reasonable, although slightly less conservative, stress values. The viscoelastic strength variation is, however, significant and must be considered.

It should be remembered that all of these effects are temperature (storage location) dependent and vary from propellant to propellant. Consequently no hard and fast rule can be set up about neglecting any of them.

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References

[1] Heller, R.A., Kamat, M.P., and Singh, M.P., "Probability of Solid Propellant Motor Failure Due to Environmental Temperatures," *Journal of Spacecraft and Rockets,* Vol. 16, May-June 1979, pp. 140-146.

[2] Christensen, R.M., *Theory of Viscoelasticity,* Academic Press, New York, 1971.

[3] Saunders, S.C., "Some Statistical Aspects of the Determination of a Safe Life from Fatigue Data," Boeing Scientific Research Laboratories, Doc. D1-82-0515, April 1966.

[4] Layton, L.H., "Chemical Aging Studies on ANB-3066 and TP-H1011 Propellants," Final Technical Report, AFRPL-TR-74-16, 1974.

[5] Cost, T.L., "Service Life Predictions for Thermal Loads Using Viscoelastic Finite Element Simulation Models," Final Report, Contract DAAH01-76-C-1069, USAMICOMT-CR-79-3, Dec. 1978.

[6] Plamgren, A., "Die Lebensdauer von Kugellagern," *VDI Zeitschrift,* Vol. 68, No. 14, 1924, p. 339.

[7] Miner, M.A., "Cumulative Damage in Fatigue," *Journal of Applied Mathematics,* Vol. 12, 1945, p. A-159.

[8] Bills, K.W. Jr. and Wiegand, J.H., "The Application of an Integrated Structural Analysis to the Prediction of Reliability," *Annals of Reliability and Maintainability,* 1970, pp. 514, 526.

[9] Heller, R.A. and Singh, M.P., "Allowable Strength of Viscoelastic Materials Under Variable Thermal Loads," *Thermal Stresses in Severe Thermal Environments,* Plenum Press, New York, 1980, pp. 491-502.

[10] Heller, R.A. and Singh, M.P., "Computer Code Using Random Probability Techniques for Rocket Motor Service Life Prediction," USAMICOM Rept. TR#RK-CR-80-1, Pt. II, May 1980.

[11] Heller, R.A. and Donat, R.C., "Fatigue Failure of a Redundant Structure," ASTM STP 404, 1964, pp. 55-66.